MATH 117: Daily Assignment 5
Selected Solutions

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August 6, 2022

Some hints for this assignment are written in the footnotes. See the daily assignment webpage for due dates, templates, and assignment description. Try to explain your reasoning and justify your computations for every problem. You should not appeal to any theorems that we have not proved yet.

1. Let $V = F^2$ where $F = \mathbb{Z}_5$. Denote the elements of $\mathbb{Z}_5$ by $0, 1, 2, 3, 4$. A basis for V is given by $B = ((1,2),(1,0))$. Define a linear map $L : V^* \to V^*$ as follows: for any linear functional $f \in V^*$, $L(f)$ is the linear functional satisfying $L(f)(a,b) = f(a+3b, 4b)$. Compute $[L]_{B^*}$ where B^* is the dual basis of B.

Solution. Let $B_* = (\varphi_1, \varphi_2)$ be the dual basis. Recall the $\varphi_1 : V \to F$ is the unique linear map satisfying $\varphi_1(1,2)) = 1$ and $\varphi_1(1,0) = 0$. Similarly, $\varphi_2 : V \to F$ is the unique linear map satisfying $\varphi_2(1,2) = 0$ and $\varphi_2(1,0) = 1$. Using Theorem 3.1.4, we have

$$[L(\varphi_1)]_{B^*} = (L(\varphi_1)(1,2), L(\varphi_1)(1,0)) = (\varphi_1(2,3), \varphi_1(1,0))$$

and

$$[L(\varphi_2)]_{B^*} = (L(\varphi_2)(1,2), L(\varphi_2)(1,0)) = (\varphi_2(2,3), \varphi_2(1,0)).$$

In order to compute, for instance, $\varphi_1(2,3)$, one must compute $[(2,3)]_B = (4,3)$. Then $\varphi_1(2,3) = 4$. Similar computations show that $\varphi_1(1,0) = 0$, $\varphi_2(2,3) = 3$, and $\varphi_2(1,0) = 1$. Hence,

$$\begin{aligned} [L]_{B^*} &= \begin{pmatrix} [L(\varphi_1)]_{B^*} & [L(\varphi_2)]_{B^*} \end{pmatrix} \\ &= \begin{pmatrix} 4 & 3 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

This is just the transpose of the matrix $[K]_B$ where $K : V \to V$ is the linear map defined by $K(a,b) = (a+3b, 4b)$. $\square$