

Equations of Lines and Planes II

Name: _____

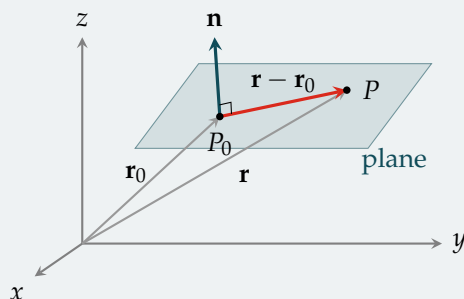
Stewart §12.5.

NORMAL VECTORS AND THE SCALAR EQUATION OF A PLANE

A plane in $\mathbb{R}^3$ is determined by a point $P_0 = (x_0, y_0, z_0)$ on the plane together with a **normal vector** denoted $\mathbf{n}$ which is orthogonal to every vector parallel to the plane. A point $P = (x, y, z)$ lies on the plane if and only if

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0,$$

where $\mathbf{r}$ and $\mathbf{r}_0$ denote the position vectors of P and P_0 , respectively.



Writing $\mathbf{n} = \langle a, b, c \rangle$ gives the **scalar equation** of the plane,

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0,$$

or equivalently $ax + by + cz = d$, where $d = ax_0 + by_0 + cz_0$.

Activity 1 _____ A plane through three points

At the board with your group. *I will assign the groups.* Roles by **first** name in **reverse** alphabetical order: first is **scribe**, second is **verifier**, third is **reporter**.

GROUP WORK — AT THE BOARD

Find your group number below.

Groups 1, 5 $A = (1, 0, 2), \quad B = (2, 1, 0), \quad C = (0, 3, 1)$

Groups 2, 6 $A = (1, 4, -1), \quad B = (0, -1, 4), \quad C = (3, -2, 1)$

Groups 3, 7 $A = (2, 0, 1), \quad B = (1, 3, 0), \quad C = (0, 2, 2)$

Groups 4, 8 $A = (1, 0, 2), \quad B = (2, 1, 0), \quad C = (0, 0, 0)$

On the board, large enough to read from across the room:

1. Sketch a picture of the plane.
2. Find a normal vector to the plane. Compute it and add it to the sketch.
3. Find an equation of the plane and write it in the form $ax + by + cz = d$.
4. Check that all 3 points lie on the plane. Is this sufficient to check if the equation is correct?

Done early? Find a fourth point on your plane, and a point that is not on your plane.

Activity 1 continued. Fill in the rest of the table during the discussion.

Groups	Equation of the plane	Same plane as Groups 1, 5?
1, 5		
2, 6		
3, 7		
4, 8		

WHEN DO TWO EQUATIONS DESCRIBE THE SAME PLANE?

Activity 2 _____ **The distance from a point to a plane**

With the person next to you. Let $Q = (3, 1, 4)$ and consider the plane $2x - y + 2z = 5$.

- (a) Find a point P_0 on the plane, a normal vector $\mathbf{n}$ to the plane, and the vector $\overrightarrow{P_0Q}$.
- (b) Sketch the plane, $\mathbf{n}$, P_0 , and Q . Explain why the distance from Q to the plane is the absolute value of the scalar projection of $\overrightarrow{P_0Q}$ onto $\mathbf{n}$.
- (c) Compute the distance from Q to the plane.

DISTANCE FROM A POINT TO A PLANE

Activity 3 The line of intersection of two planes

Complete this activity at the board with your same group. **Rotate roles:** the **verifier** is now scribe, the **reporter** is now verifier, and the **scribe** is now reporter.

GROUP WORK — AT THE BOARD

Find your group number below.

Groups 1, 5 $5x + 3y + 4z = 13$ and $x + y + z = 3$

Groups 2, 6 $20x + 12y + 16z = 52$ and $z = 2$

Groups 3, 7 $5x + 3y + 4z = 14$ and $x = 2$

Groups 4, 8 $-2x + 4y + z = 0$ and $2x + y - z = 0$

It might be helpful to sketch a generic picture of two intersecting planes.

1. Show that the planes are not parallel.
2. Find a point of intersection.
3. Find the direction of the line of intersection.
4. Find vector or parametric equations for the line (add it to the picture).
5. Find the plane containing the point $(1, 1, 1)$ that is perpendicular to both planes.

Done early? Compare your line to the line $x - 1 = \frac{z + 1}{2}, y = 0$. Are they parallel, intersecting, or skew? Explain.

Activity 4 _____ The plane between two skew lines

Complete this activity at the board with your same group. **Rotate roles:** the **verifier** is now scribe, the **reporter** is now verifier, and the **scribe** is now reporter.

GROUP WORK — AT THE BOARD

Find your group number below.

Groups 1, 5 $L_1: \langle 1, 0, 0 \rangle + t\langle 1, 1, 0 \rangle, \quad L_2: \langle 0, 1, 4 \rangle + s\langle 0, 1, 1 \rangle$

Groups 2, 6 $L_1: \langle 0, 1, 0 \rangle + t\langle 1, 0, 2 \rangle, \quad L_2: \langle 1, 0, 1 \rangle + s\langle 0, 1, -1 \rangle$

Groups 3, 7 $L_1: \langle 1, 1, 0 \rangle + t\langle 2, 1, 0 \rangle, \quad L_2: \langle 0, 0, 5 \rangle + s\langle 1, 0, 1 \rangle$

Groups 4, 8 $L_1: \langle 2, 0, 1 \rangle + t\langle 1, -1, 1 \rangle, \quad L_2: \langle 0, 2, 0 \rangle + s\langle 1, 1, 0 \rangle$

It might be helpful to sketch a generic picture of two skew lines: draw two parallel planes, one above the other, with one line in each plane pointing in a different direction.

1. Show that the lines are skew.
2. Find a normal vector to a plane that is parallel to both lines.
3. Find the plane containing L_1 that is parallel to L_2 , and the plane containing L_2 that is parallel to L_1 (add them to the picture).
4. Find the plane that is parallel to both lines and equidistant from them (add it to the picture). *Hint:* use the midpoint of a point on L_1 and a point on L_2 .

Exit ticket

Name: _____

Complete this problem on your own. You must turn this sheet in at the end of class; it counts toward Engagement.

Find an equation of the plane through $(1, 0, 2)$ that is perpendicular to the line $x = 3 + 2t$, $y = -t$, $z = 4 + t$. Then find the point where the line meets the plane. It might be helpful to sketch a picture.