Three-Dimensional Coordinate Systems

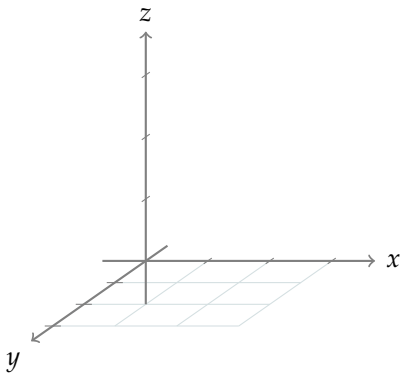
Name: _____

Stewart §12.1. Today's question: *what does an equation like $x = 2$ actually mean?*

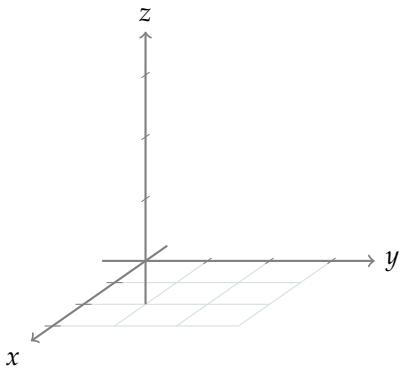
Keep Worksheet 1 out — we finish Activities 3 and 4 first.

Warm-up: whose picture is right?

Maya and Theo each drew a set of axes for three-dimensional space, and labeled them like this:



Theo



Maya

(a) Plot the point $(1, 2, 3)$ on **both** sets of axes, using each person's labels. What do you notice?

(b) So the ordered triple $(1, 2, 3)$ does not, by itself, determine a location in space. What else has to be agreed on first?

(c) Point your **right** hand in the direction of the positive x-axis and curl your fingers toward the positive y-axis. Where does your thumb point? Try it on each picture above and draw an arrow on each picture showing the direction your thumb points.

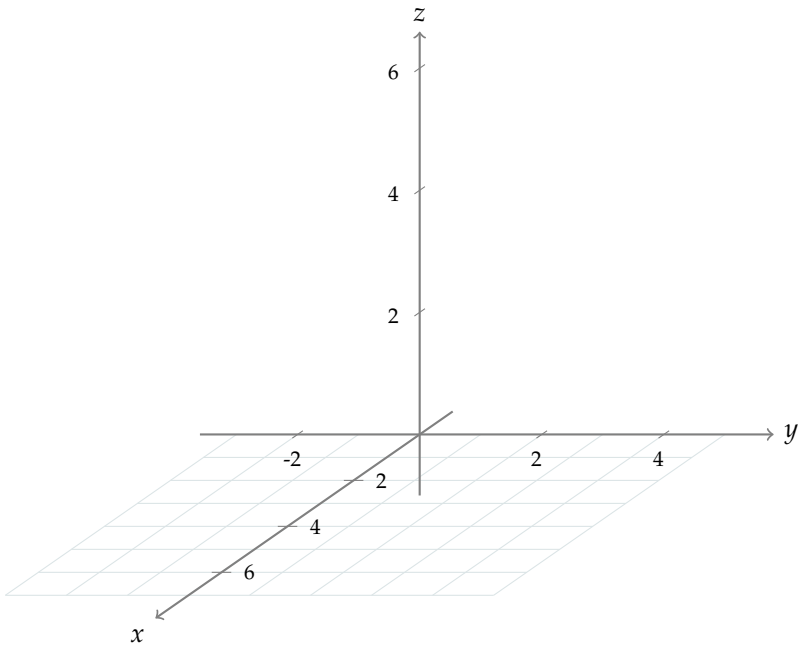
NOTATION, THE RIGHT-HAND RULE, PLANES, OCTANTS

Activity 1 — Getting around in space[1]

Throughout, *forward* means the positive x-direction, *right* means positive y, and *up* means positive z.

(a) Start at $(1, 2, 3)$. Move 5 units forward, 4 units left, and 2 units up. Where do you end up? Call this point A.

(b) Plot A on the axes below. Draw the three segments parallel to the axes that show how to get there from the origin — along x, then y, then up.



(c) On your sketch, find the point of the xy-plane that is *closest* to A. What are its coordinates? Now do the same for the xz-plane and the yz-plane.

(d) Compare each of those three points to A itself. What is the rule? This point is called the **projection** of A onto the plane.

(e) Which coordinate plane is A closest to? How far is it from that plane? Can you answer without computing anything?

[1] Activities 1 and 3 adapted from *Active Calculus Multivariable* §9.2 (Boelkins et al.), CC BY-SA 4.0.

Activity 2 — One equation, three different spaces

Complete the next activity in groups at the board. *I will assign the groups.* Roles are determined by first name in **reverse** alphabetical order: first is **scribe**, second is **verifier**, third is **reporter**. You are free to negotiate roles with your group.

GROUP WORK — AT THE BOARD

Find your group number below.

Groups 1, 5	$x = 2$
Groups 2, 6	$z = 3$
Groups 3, 7	$y = x$
Groups 4, 8	$x^2 + y^2 = 1$

For your equation, draw **three** pictures side by side and describe it in words.

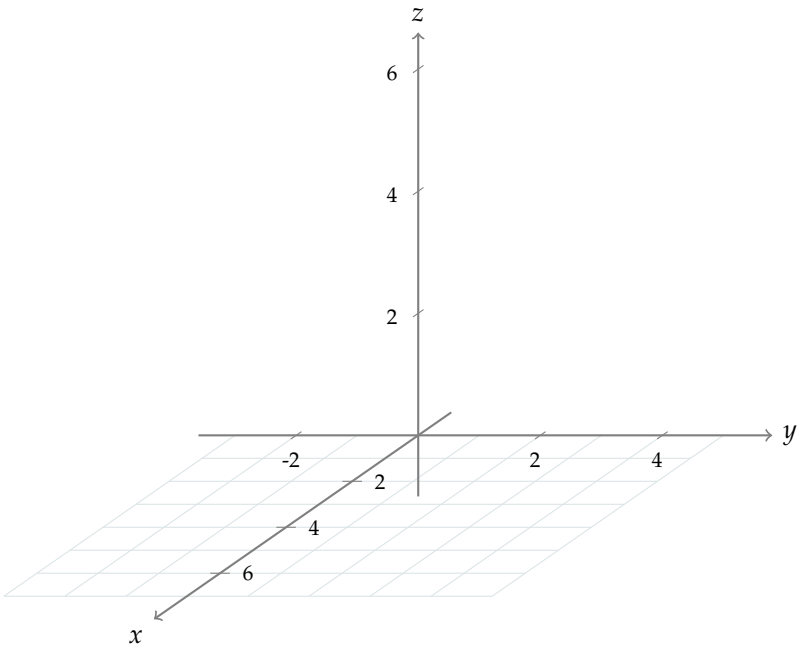
1. the set of points satisfying it on the number line $\mathbb{R}$ (if it makes sense there);
2. the set of points satisfying it in the Cartesian plane $\mathbb{R}^2$;
3. the set of points satisfying it in 3-dimensional space $\mathbb{R}^3$.

Done early? What is the set of points in $\mathbb{R}^3$ satisfying *both* $x^2 + y^2 = 1$ *and* $z = 3$?

Activity 2 continued. Fill in the rest of the table during the discussion.

Groups	Equation	In $\mathbb{R}$	In $\mathbb{R}^2$	In $\mathbb{R}^3$
1, 5	$x = 2$			
2, 6	$z = 3$			
3, 7	$y = x$			
4, 8	$x^2 + y^2 = 1$			

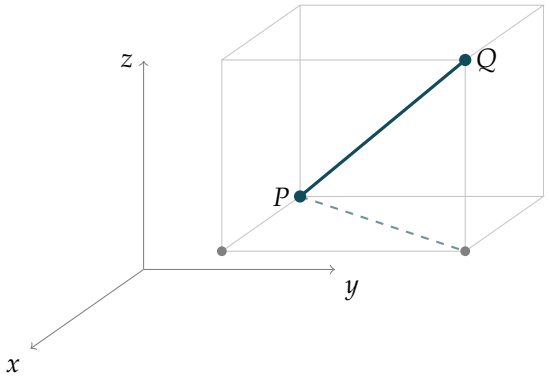
Sketch your own group's $\mathbb{R}^3$ picture here.



WHAT AN EQUATION MEANS DEPENDS ON THE SPACE

Activity 3 — How far apart are two points?

Let $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$. They are opposite corners of a rectangular box whose faces are parallel to the coordinate planes.



(a) Two other corners of the box are marked. Label the one at the end of the dashed segment R, and label the remaining marked corner S. Write down the coordinates of each in terms of $x_1, y_1, z_1, x_2, y_2, z_2$.

(b) Which coordinate do P, R, and S all share? Give the equation of the plane containing the triangle PRS.

(c) Triangle PRS has its right angle at S. Find $|PS|$ and $|SR|$, then use the Pythagorean Theorem to find $|PR|$ (the dashed segment).

(d) Triangle PRQ has its right angle at R. What is $|RQ|$? Apply the Pythagorean Theorem a second time to find $|PQ|$.

(e) Find the distance between $(4, 5, 6)$ and $(3, 7, 8)$.

DISTANCE, AND THE EQUATION OF A SPHERE

Activity 4 — One more

Find an equation of the sphere centered at $(4,5,6)$ that passes through the point $(3,7,8)$. *You already did most of the work in Activity 3(e).*