

Partial Derivatives

Name: _____

Stewart §14.3.

Warm-up _____ Wind chill¹

With the person next to you. The wind chill w , in degrees Fahrenheit, measures how cold it feels outside when the wind is blowing. It is a function of the wind speed v , in miles per hour, and the air temperature T , in degrees Fahrenheit, so $w = w(v, T)$.

$v \backslash T$	-30	-25	-20	-15	-10	-5	0	5	10	15	20
5	-46	-40	-34	-28	-22	-16	-11	-5	1	7	13
10	-53	-47	-41	-35	-28	-22	-16	-10	-4	3	9
15	-58	-51	-45	-39	-32	-26	-19	-13	-7	0	6
20	-61	-55	-48	-42	-35	-29	-22	-15	-9	-2	4
25	-64	-58	-51	-44	-37	-31	-24	-17	-11	-4	3
30	-67	-60	-53	-46	-39	-33	-26	-19	-12	-5	1
35	-69	-62	-55	-48	-41	-34	-27	-21	-14	-7	0
40	-71	-64	-57	-50	-43	-36	-29	-22	-15	-8	-1

- (a) At a fixed temperature of -10°F , find the average rate of change of w with respect to v as the wind speed increases from 15 mi/h to 25 mi/h. Include units. What does the number mean in words?
- (b) At a fixed temperature of -10°F , find the average rate of change of w with respect to v as the wind speed increases from 15 mi/h to 20 mi/h. Include units. What does the number mean in words?
- (c) Which of (a) or (b) better approximates the instantaneous rate of change of w with respect to v when $v = 15$ mi/h and $T = -10^\circ\text{F}$? What would you need in order to get a better approximation?

¹The table is from Activity 10.2.5 of *Active Calculus — Multivariable* (Schlicker, Keller, Long), activecalculus.org, CC BY-SA 4.0.

RATES OF CHANGE OF TRACES

FIRST-ORDER PARTIAL DERIVATIVES

The **partial derivatives** of f with respect to x and y at (a, b) are

$$\frac{\partial f}{\partial x}(a, b) = f_x(a, b) = \lim_{h \rightarrow 0} \frac{f(a + h, b) - f(a, b)}{h},$$

$$\frac{\partial f}{\partial y}(a, b) = f_y(a, b) = \lim_{h \rightarrow 0} \frac{f(a, b + h) - f(a, b)}{h},$$

provided the limits exist.

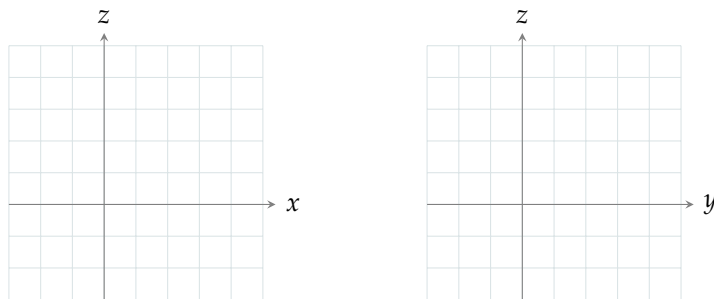
COMPUTING PARTIAL DERIVATIVES

Activity 1 _____ Partial derivatives and traces²

With your group. Consider the function

$$f(x, y) = \frac{xy^2}{x+1}$$

at the point $(1, 2)$.



Left: the $y = 2$ trace $z = f(x, 2)$. Right: the $x = 1$ trace $z = f(1, y)$.

- (a) Draw the $y = 2$ trace $z = f(x, 2)$ on the left above. Also sketch the tangent line at $x = 1$.
- (b) Find the partial derivative $f_x(1, 2)$ and relate its value to your sketch.
- (c) Draw the $x = 1$ trace $z = f(1, y)$ on the right above. Also sketch the tangent line at $y = 2$.
- (d) Find the partial derivative $f_y(1, 2)$ and relate its value to your sketch.

²Adapted from Activity 10.2.2 of *Active Calculus — Multivariable* (Schlicker, Keller, Long), activecalculus.org, CC BY-SA 4.0.

Activity 2 _____ Computing partial derivatives³

Together as a class.

(a) If $f(x, y) = 3x^3 - 2x^2y^5$, find f_x and f_y .

(b) If $g(r, s) = rs \cos(r)$, find g_r and g_s .

(c) If $f(w, x, y) = (6w + 1) \cos(3x^2 + 4xy^3 + y)$, find f_w , f_x , and f_y .

³Adapted from Activity 10.2.3 of *Active Calculus — Multivariable* (Schlicker, Keller, Long), activecalculus.org, CC BY-SA 4.0.

Activity 3 _____ Back to the wind chill⁴

With your group. Use the table from the warm-up. From warm-up (b), $w_v(15, -10) \approx -0.6$ °F per mi/h.

- (a) Estimate $w_T(15, -10)$. What are the units on this quantity, and what does it mean?
- (b) Use your results to estimate $w(13, -10)$. (Recall that for a function f of one variable, $f(x + h) \approx f(x) + hf'(x)$.)
- (c) Use your results to estimate $w(15, -12)$.
- (d) How might you combine your previous results to estimate $w(13, -12)$? Explain your process.

ESTIMATING AND APPROXIMATING WITH PARTIAL DERIVATIVES

⁴Adapted from Activity 10.2.5 of *Active Calculus — Multivariable* (Schlicker, Keller, Long), activecalculus.org, CC BY-SA 4.0.

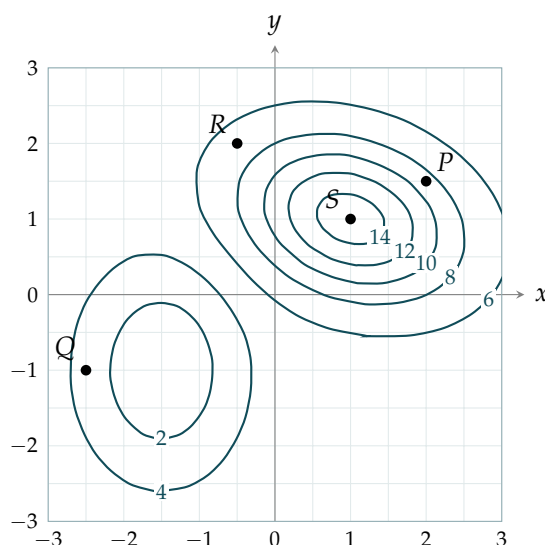
Exit ticket

Name: _____

Complete this and turn it in at the end of class; it counts toward Engagement. **You may discuss with the people around you.**

- (a) Let $f(x, y) = x^2y - y^3$. Find $f_x(1, 2)$ and $f_y(1, 2)$. Standing on the graph of f at $(1, 2, -6)$, are you going uphill or downhill if you walk in the positive x -direction? In the positive y -direction?

- (b) Below is a contour map of the elevation $g(x, y)$ of a landscape with a peak and a valley. The contours are drawn at levels 2, 4, 6, $\dots$, 14.



- (i) At each of P , Q , R , and S , is g_x positive, negative, or zero? Is g_y ?

- (ii) Estimate $g_x(2, 1.5)$ and $g_y(2, 1.5)$.