

Functions of Several Variables

Name: _____

Stewart §14.1.

FUNCTIONS OF TWO VARIABLES

A **function of two variables** f is a rule that assigns to each ordered pair (x, y) in a set $D \subseteq \mathbb{R}^2$ a unique real number $f(x, y)$. The set D is the **domain** of f , and the **range** of f is the set of values it takes, $\{f(x, y) : (x, y) \in D\}$. When f is given by a formula and no domain is specified, the domain is the set of all (x, y) for which the formula makes sense.

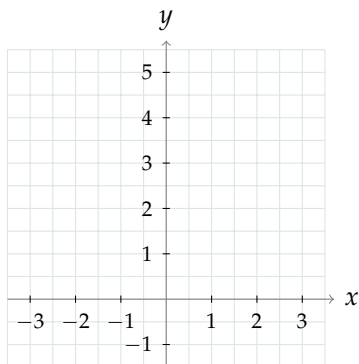
Activity 1 _____ Domain and range

Find the domain and range of each function.

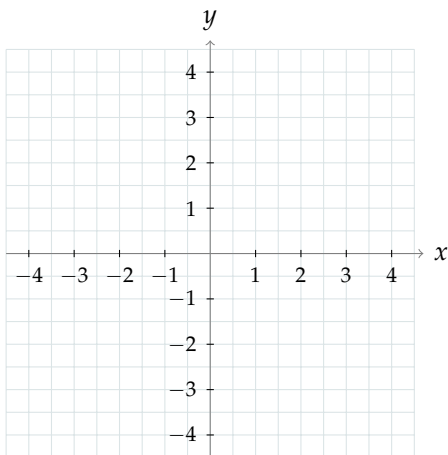
(a) $f(x, y) = x^2 + y^2$

(b) $f(x, y) = \sqrt{x^2 + y^2}$

(c) Let $f(x, y) = \frac{\sqrt{y - x^2}}{x - 2}$. Evaluate $f(1, 3)$, then find and sketch the domain of f .



(d) Let $g(x, y) = \ln(9 - x^2 - y^2)$. Find and sketch the domain of g . What is its range?



THE GRAPH

The **graph** of f is the surface $\{(x, y, z) \in \mathbb{R}^3 : z = f(x, y), (x, y) \in D\}$.

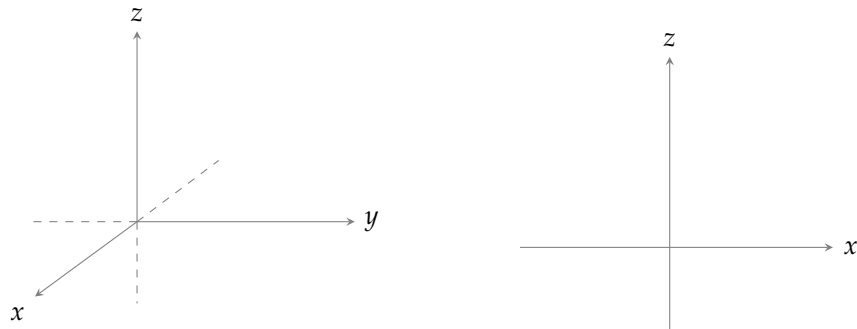
TRACES

A **trace** of a function $f(x, y)$ in the x direction is a curve defined by the equation

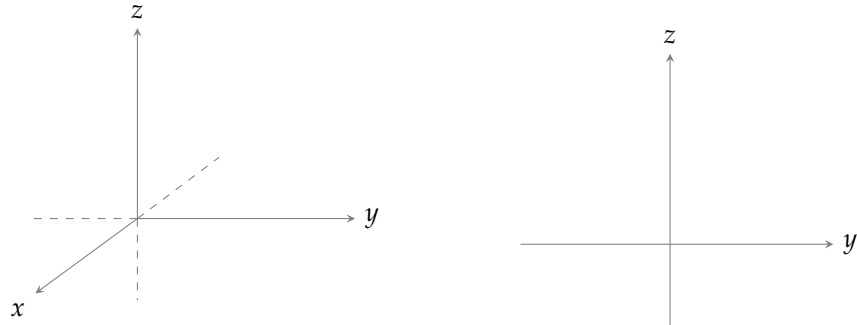
$$z = f(x, c)$$

for some constant $c \in \mathbb{R}$. Similarly, a trace of f in the y direction is a curve of the form $z = f(c, y)$.

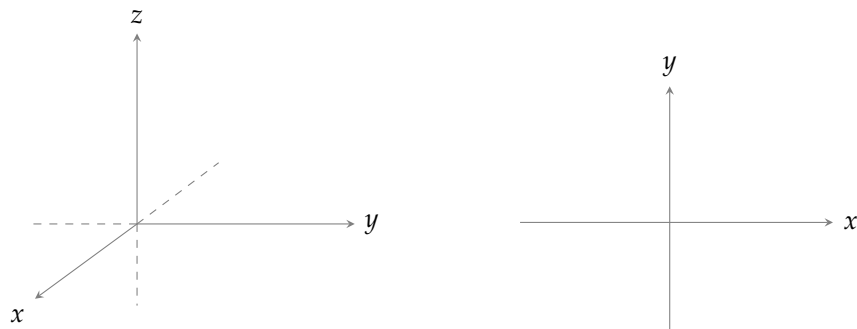
Trace in the x -direction: intersect the graph of $f(x, y)$ with the plane $y = c$. Draw the slice in the xz -plane.



Trace in the y -direction: intersect the graph with the plane $x = c$. Draw in the yz -plane.

**LEVEL CURVES**

The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f). A **contour map** is a collection of level curves, labeled by their values of k .



Activity 2 Building surfaces from level curves

At the board with your group. *I will assign the groups.* Roles by **first** name in **alphabetical** order: first is **scribe**, second is **verifier**, third is **reporter**.

GROUP WORK — AT THE BOARD

Find your group number below.

Groups 1, 5	$f(x, y) = \sqrt{16 - x^2 - y^2}$	$k = 0, 1, 2, 3, 4$
Groups 2, 6	$f(x, y) = y^2 - x^2$	$k = -2, -1, 0, 1, 2$
Groups 3, 7	$f(x, y) = \sqrt{x^2 + y^2}$	$k = 0, 1, 2, 3$
Groups 4, 8	$f(x, y) = y - x^2$	$k = -1, 0, 1, 2$

On the board, large enough to read from across the room:

1. Sketch the level curves for your values of k in the xy -plane. Label each curve with the corresponding k value.
2. As k increases, do the curves get closer together, farther apart, or evenly spaced?
3. Find the traces in the x direction for several values of c . Sketch the traces in the yz -plane.
4. Find the traces in the y direction for several values of c . Sketch the traces in the xz -plane.
5. Using your traces and level curves, try to sketch the graph of f . What does it look like?

Done early? Find a vector-valued function $\mathbf{r}(t)$ which traces out your trace $z = f(x, 1)$. Find $\mathbf{r}'(t)$. What do you think $\mathbf{r}'(t)$ tells you about the function?

Groups	Level curves	Spacing as k increases	Surface
1, 5			
2, 6			
3, 7			
4, 8			

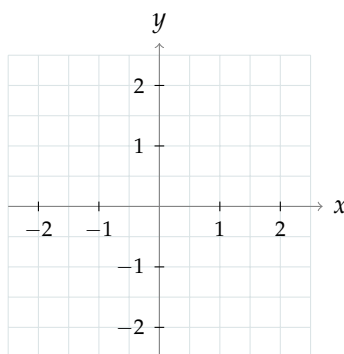
WHAT DOES THE SPACING OF LEVEL CURVES TELL YOU?

Activity 3 _____ Reading a table of values

With the person next to you. The table gives values of a function $f(x, y)$.

$y \backslash x$	-2	-1	0	1	2
2	-4	-2	0	2	4
1	-2	-1	0	1	2
0	0	0	0	0	0
-1	2	1	0	-1	-2
-2	4	2	0	-2	-4

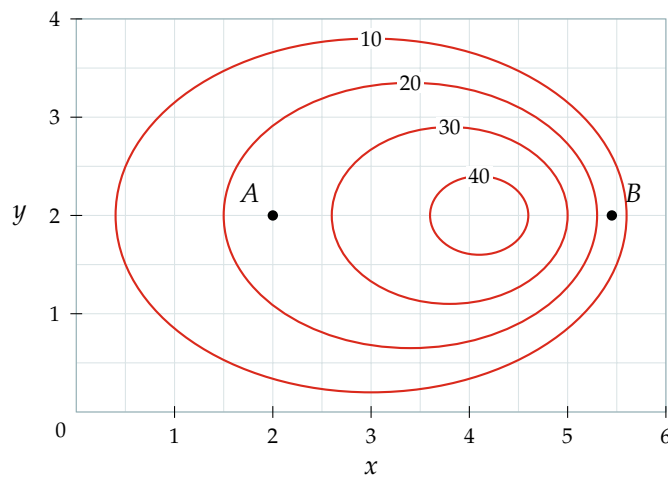
- (a) What is $f(1, -2)$?
- (b) Find every point in the table where $f = 2$ and plot them. What curve do they seem to lie on? Do the same for $f = 0$.



- (c) Read across the row $y = 1$. How does f change as x increases? What trace of the graph of f is this row?
- (d) Read down the column $x = -1$. How does f change as y increases? What trace of the graph of f is this column?
- (e) Guess a formula for f .

Activity 4 _____ Estimating from a contour map

With the person next to you. The contour map below shows the height $f(x, y)$, in meters, of a hill.



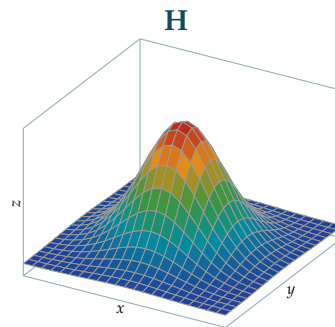
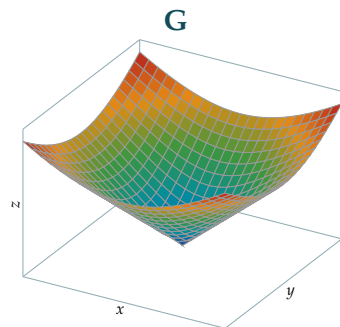
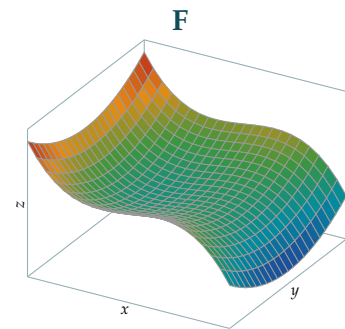
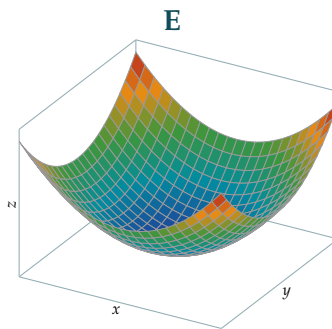
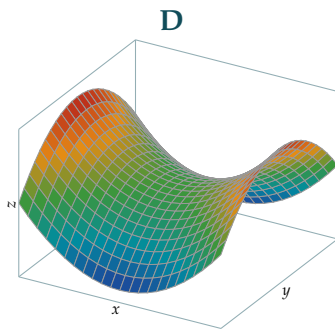
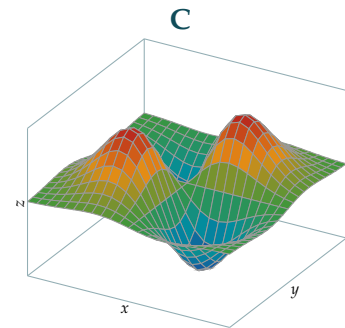
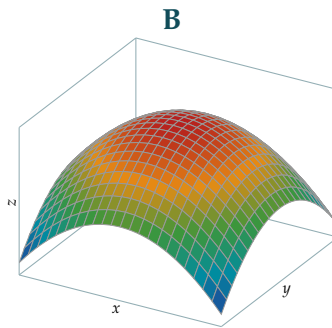
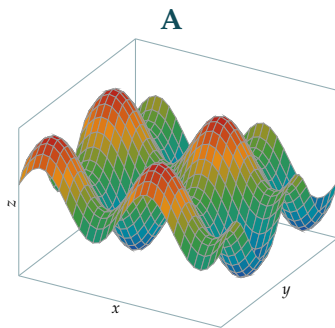
- (a) Estimate the height at A and at B . Is $f(4, 3)$ more or less than 30?
- (b) Which side of the hill is steeper, the east (right) or the west (left)? How can you tell without computing anything?
- (c) You walk along the curve labeled 20. What happens to your height?

Exit ticket

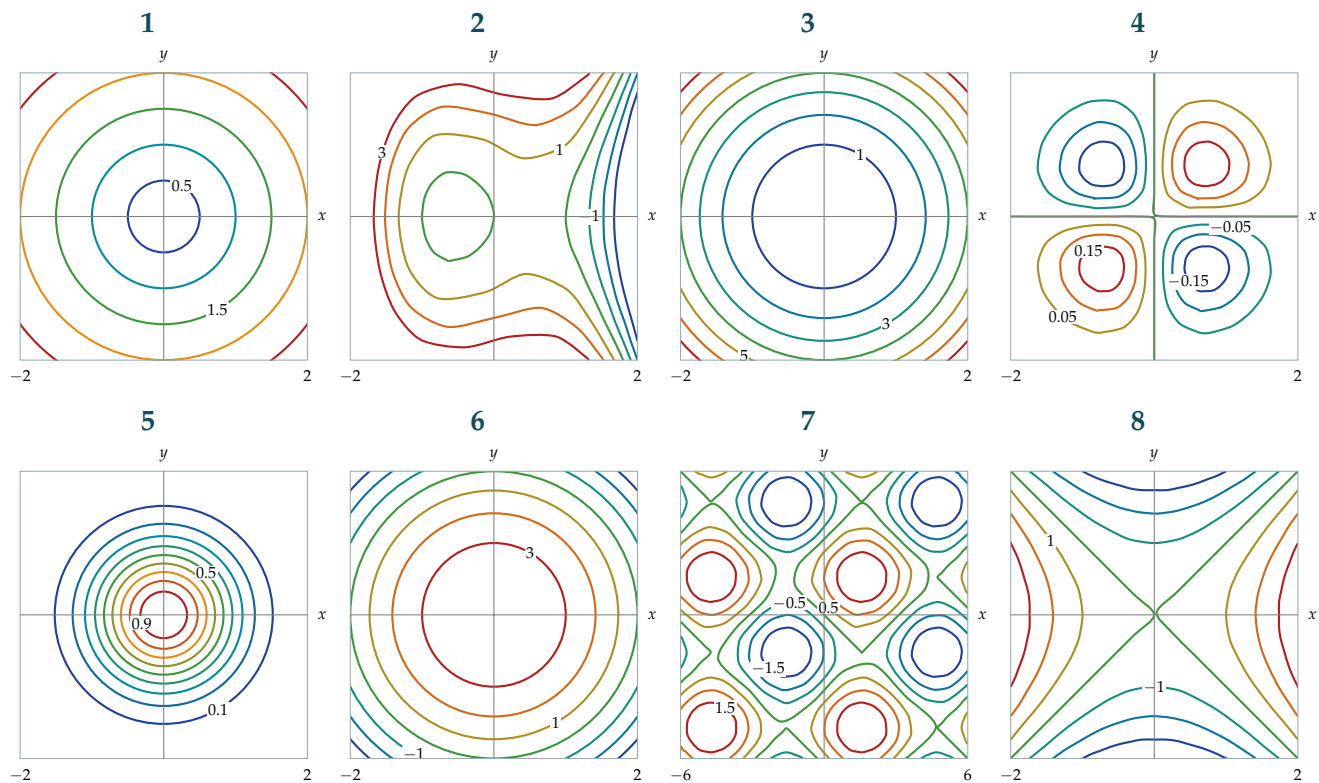
Name: _____

Complete this and turn it in at the end of class; it counts toward Engagement. **You may discuss with the people around you.**

The graphs of eight functions are plotted below.¹ Their contour maps are plotted on the next page. Each level curve is colored by its elevation, from blue (lowest) to red (highest). Match each graph to its contour map.



¹Borrowed from the gallery of functions §9.1 of *Active Calculus: Multivariable*. CC BY-SA 4.0.



(a) Match each graph to the corresponding contour map.

A	B	C	D	E	F	G	H

(b) Contour maps 3 and 6 have the same level curves. Explain how you can tell them apart.

(c) On map 5, where is graph H steepest? How can you tell from the map alone?

(d) Using only map 8, describe the trace of graph D along the x -axis and along the y -axis.