Curves Defined by Parametric Equations

Name: _____

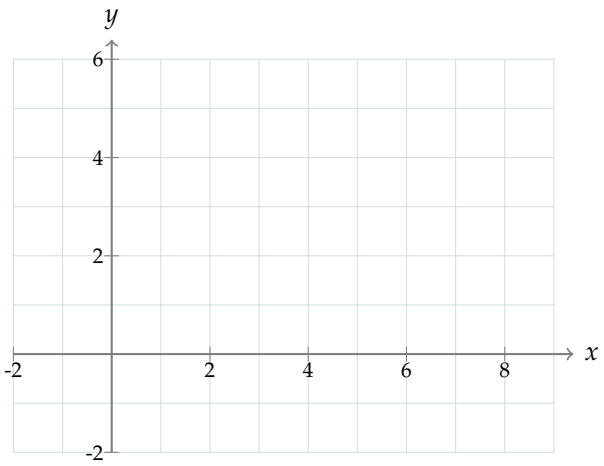
Stewart §10.1. Today's question: *what is the difference between a circle and driving around a circle?*

Warm-up: a curve that isn't a graph

Let $x = t^2 - 2t$ and $y = t + 1$.

(a) Fill in the table. Then plot each point **and label it with its t-value**. Do not connect them yet.

t	x	y
-2		
-1		
0		
1		
2		
3		
4		



(b) Draw a smooth curve through the points and put an arrow on the curve showing the direction of travel as the values of t *increase*.

(c) The t-values are equally spaced. Are the points equally spaced on the page?

(d) Imagine that the curve traces out the path of a person walking around the park at time t. When is the person moving fastest? Slowest?

(e) Does the person ever stop moving?

DEFINITIONS

Activity 1 — What does eliminating the parameter cost?

(a) Solve $y = t + 1$ for t, substitute into $x = t^2 - 2t$, and simplify to get a Cartesian equation relating x and y alone.

(b) Your equation from (a) and the parametric equations describe the same set of points. Name **two** things the parametric equations tell you that the Cartesian equation does not.

In summary: the Cartesian equation describes the road; the parametric equations describes the drive.

Activity 2 — Different drives, same road?

For the next activity, we will work together in groups at the board. *I will assign the groups.* Group roles are determined by first name alphabetically: first is **scribe**, second is **verifier**, third is **reporter**.

GROUP WORK — AT THE BOARD

Find your group number below. All four are on $0 \le t \le 2\pi$.

Groups 1, 5	$x = \cos t, \quad y = \sin t$
Groups 2, 6	$x = \sin 2t, \quad y = \cos 2t$
Groups 3, 7	$x = \cos(t + \pi), \quad y = \sin(t + \pi)$
Groups 4, 8	$x = \cos\left(\frac{t}{2}\right), \quad y = \sin\left(\frac{t}{2}\right)$

Two groups have each set of equations. You and your twin group should end up with the same picture — if you don't, that is the interesting part.

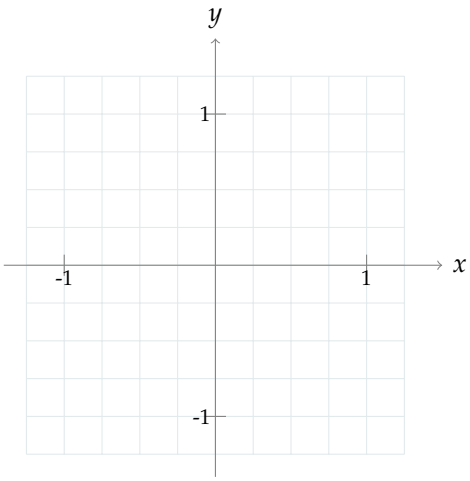
On the board, large enough to read from across the room:

1. a table of t, x, y for $t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots, 2\pi$ (recall $\frac{\sqrt{2}}{2} \approx 0.7$);
2. the plotted curve, with the t-value written beside each point;
3. an arrow showing the direction of travel;
4. your **initial** and **terminal** points, circled.

Done early? Write down a *fifth* set of parametric equations that traces the same curve, but starts at $(0, -1)$ and goes clockwise. Put it on the board next to your work.

Activity 2 continued.

Sketch your own groups curve on the axis below, then fill the rest of the table during the discussion.



Groups	Equations	Curve	Starts at	Direction	Times around
1, 5	$\cos t,\ \sin t$				
2, 6	$\sin 2t,\ \cos 2t$				
3, 7	$\cos(t+\pi),\ \sin(t+\pi)$				
4, 8	$\cos(t/2),\ \sin(t/2)$				

Every group's x and y satisfy the same Cartesian equation. What is it?

CURVE VS. PARAMETRIC CURVE

Activity 3 — Modifying curves

We will graph $x = \cos t$, $y = \sin t$ on $0 \le t \le 2\pi$, modified by some constant c, using graphing software like GeoGebra (or Desmos). What do you think will happen as we change the value of c? **Predict first**, then we drag the slider and check.

Parametric equations	Your prediction	What happens
$x = \cos t,\ y = \sin t + c$		
$x = \cos t - c,\ y = \sin t$		
$x = c\cos t,\ y = \sin t$		
$x = c\cos t,\ y = c\sin t$		
$x = \cos(ct),\ y = \sin(ct)$		

Activity 4 — Parameterizing circles

Complete this activity at the board with your same group. **Rotate roles:** the **verifier** is now scribe, the **reporter** is now verifier, and the **scribe** is now reporter.

GROUP WORK — AT THE BOARD

Using your observations from Activity 3:

(a) Write parametric equations for a circle of radius 5 centered at $(2, -3)$. Give the interval of t as well. What is the initial point of your parametric curve, and which direction does it go?

(b) Modify your equations so that the circle **starts at** $(2, -8)$ and is traced **clockwise**.

Leave both sets of equations on the board. There is more than one right answer to (b) — we will compare.

Record your group's answer to (b) here, along with any *different* answer from another group that also works. If every group wrote the same thing, find one more that works and add it yourself.

Unit circle reference