

## Limits

Name: \_\_\_\_\_

Preview Activity for Monday 9/28 · Submit via Gradescope by 11:59 p.m. Sunday 9/27.

**Review:** We say that a function  $f$  has a limit  $L$  as  $x$  approaches  $a$  provided that we can make the values  $f(x)$  as close to  $L$  as we like by taking  $x$  sufficiently close (but not equal) to  $a$ . We denote this behavior by writing

$$\lim_{x \rightarrow a} f(x) = L.$$

Activity \_\_\_\_\_ Limits from tables and graphs<sup>1</sup>

We investigate the limits of several different functions by working with tables and graphs.

- (a) Consider the function  $f$  defined by

$$f(x) = 3 - x.$$

Complete the table below.

|        |      |      |     |     |     |
|--------|------|------|-----|-----|-----|
| $x$    | -0.2 | -0.1 | 0.0 | 0.1 | 0.2 |
| $f(x)$ |      |      |     |     |     |

What does the table suggest regarding  $\lim_{x \rightarrow 0} f(x)$ ?

- (b) Explain how your results in (a) are reflected in Figure 1.

- (c) Next, consider

$$g(x) = \frac{x}{|x|}.$$

Complete the table below with values near  $x = 0$ , the point at which  $g$  is not defined.

|        |      |       |        |       |      |     |
|--------|------|-------|--------|-------|------|-----|
| $x$    | -0.1 | -0.01 | -0.001 | 0.001 | 0.01 | 0.1 |
| $g(x)$ |      |       |        |       |      |     |

What does this suggest about  $\lim_{x \rightarrow 0} g(x)$ ?

- (d) Explain how your results in (c) are reflected in Figure 2.

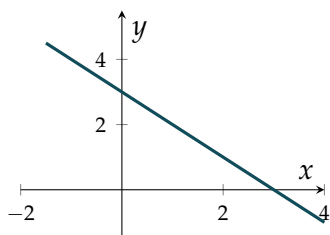


Figure 1. The graph of  $f(x) = 3 - x$ .

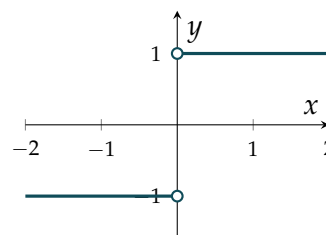


Figure 2. The graph of  $g(x) = \frac{x}{|x|}$ .

(e) Now, let's examine a function of two variables. Let

$$f(x, y) = 3 - x - 2y.$$

Complete the table below.

| $x \backslash y$ | -1.0 | -0.1 | 0.0 | 0.1 | 1.0 |
|------------------|------|------|-----|-----|-----|
| -1.0             |      | 4.2  |     |     |     |
| -0.1             |      |      |     |     | 1.1 |
| 0.0              |      |      |     | 2.8 |     |
| 0.1              | 4.9  |      |     |     |     |
| 1.0              |      |      | 2.0 |     |     |

What does the table suggest about  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ ?

(f) Explain how your results in (e) are reflected in Figure 3. Compare this limit to the limit in part (a). How are the limits similar and how are they different?

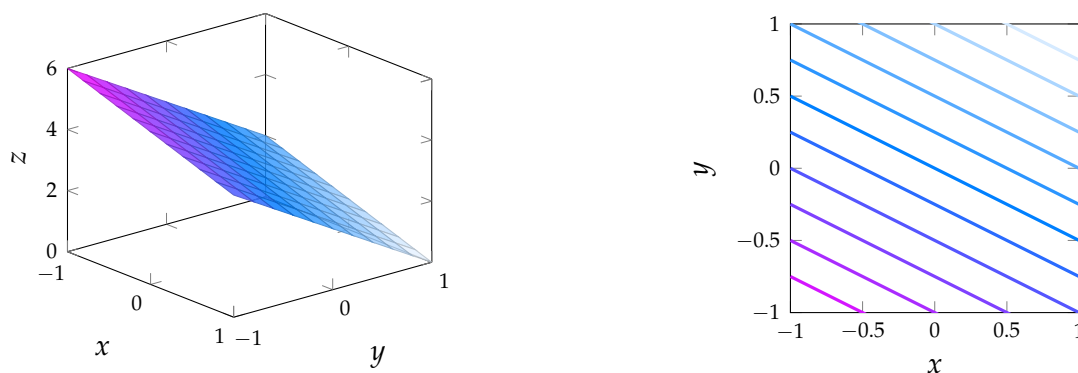


Figure 3. Left: The graph of  $f(x, y) = 3 - x - 2y$  (GeoGebra: [interactive version](#)). Right: A contour plot (level curves  $f = 0.5, 1, 1.5, \dots, 5.5$ ).

The GeoGebra version of Figure 3 shows the graph of  $f$  together with these level curves, each drawn at its height on the surface and again on the floor of the box, colored as in the contour plot.

(g) Finally, consider

$$g(x, y) = \frac{2xy}{x^2 + y^2},$$

which is not defined at  $(0, 0)$ . Complete the table below. Round to three decimal places.

| $x \backslash y$ | -1.0   | -0.1  | 0.0   | 0.1   | 1.0    |
|------------------|--------|-------|-------|-------|--------|
| -1.0             |        | 0.198 |       |       |        |
| -0.1             |        |       |       |       | -0.198 |
| 0.0              |        |       | —     | 0.000 |        |
| 0.1              | -0.198 |       |       |       |        |
| 1.0              |        |       | 0.000 |       |        |

What does this suggest about  $\lim_{(x,y) \rightarrow (0,0)} g(x, y)$ ?

(h) Explain how your results are reflected in Figure 4. Compare this limit to the limit in part (c). How are the results similar and how are they different?

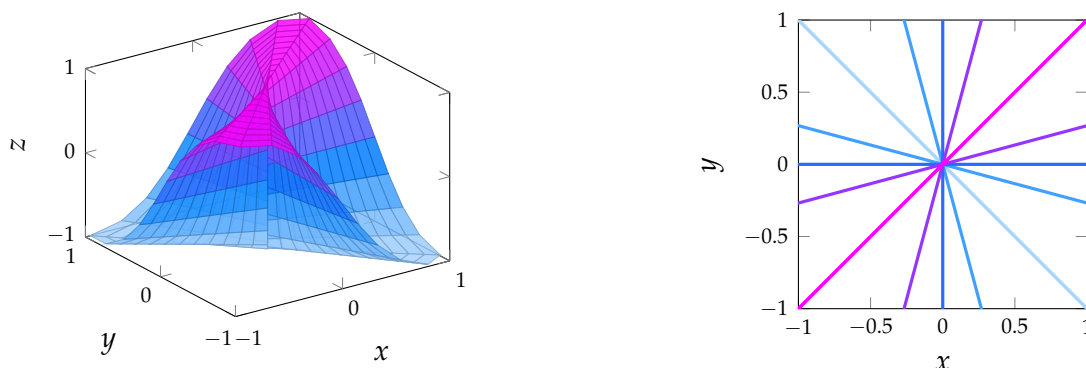


Figure 4. Left: The graph of  $g(x, y) = \frac{2xy}{x^2 + y^2}$  (GeoGebra: [interactive version](#)). Right: A contour plot (level curves  $g = -1, -0.5, 0, 0.5, 1$ ).

The GeoGebra version of Figure 4 shows the graph of  $g$  together with these level curves, each drawn at its height on the surface and again on the floor of the box, colored as in the contour plot.

(i) Let  $\mathbf{r}_1(t) = \langle t, 0 \rangle$  and  $\mathbf{r}_2(t) = \langle t, t \rangle$ . These parameterize the  $x$ -axis and the line  $y = x$ , and both pass through  $(0, 0)$  when  $t = 0$ . Simplify  $g(\mathbf{r}_1(t))$  and  $g(\mathbf{r}_2(t))$  for  $t \neq 0$ , and then find

$$\lim_{t \rightarrow 0} g(\mathbf{r}_1(t)) \quad \text{and} \quad \lim_{t \rightarrow 0} g(\mathbf{r}_2(t)).$$

These are the values  $g(x, y)$  approaches as  $(x, y)$  approaches  $(0, 0)$  along each line. How does this fit with your answer in (g)?

## Optional Reading

Stewart §14.2.

<sup>1</sup>Preview Activity 10.1.1 of *Active Calculus — Multivariable* (Schlicker, Keller, Long), [activecalculus.org](http://activecalculus.org), CC BY-SA 4.0. Part (i) is not part of the original activity.