

Midterm Exam 1 Info

Topics: The midterm exam on Friday, September 25 will cover Sections 10.1, 12.1–12.5, 13.1, 13.2, and 13.4 of Stewart. Section 13.3 (arc length and curvature) and projectile motion will not be on the exam. Anything covered in the textbook sections, the preview activities, the worksheets, or discussed in class is fair game. However, topics explicitly discussed in class will be prioritized.

What to expect: You will have the entire class period to work on the exam. Expect a mix of question types: computational problems in the style of the quizzes, short questions worth only a point or two, true/false questions requiring no justification, questions that ask you to work with a picture (sketching a vector, reading a diagram, or graphing a parametric curve), and questions that ask you to explain your reasoning (for instance, why the cross product gives an area, or what a projection measures geometrically). The worksheets are designed for learning new ideas and WebAssign is designed for practice with instant feedback, while the exam is designed to measure the learning objectives listed below. For this reason, the exam questions may look different from both. Keep in mind that no notes or calculators are allowed on the exam, so you will need to be able to recall some formulas.

Suggestion for studying: Reflect on each item below. Can you do it right now, without looking at your notes? Test yourself. Find a problem of that type and try to solve it: the exercises at the end of each section of the textbook are organized by topic, and the odd-numbered ones have the answers in the back. Try the quiz problems again without looking at the solutions, and try the worksheet problems again without any of the scaffolding (they are usually broken down into steps for you).

Here is my (non-exhaustive) summary of the specific things you should know and be able to do from each section of the book.

1. Parametric curves (10.1):

- (a) Know what parametric equations and a parametric curve are, including the initial and terminal points on an interval $a \leq t \leq b$.
- (b) Sketch a parametric curve by plotting points, and indicate the direction of travel as t increases.
- (c) Eliminate the parameter to find a Cartesian equation for the curve.
- (d) Explain the information a parametrization contains that its Cartesian equation does not, such as the direction of travel, the speed, the starting and ending points, and the number of times the curve is traced.
- (e) Find parametric equations for a circle with a given center and radius, starting at a specified point and traced in a specified direction.

2. Three-dimensional coordinates (12.1):

- (a) Plot points in $\mathbb{R}^3$ using a right-handed coordinate system. Know the coordinate planes.
- (b) Find the projection of a point onto each coordinate plane, and its distance to each coordinate plane.
- (c) Describe and draw the set of points satisfying a given equation such as $x = 2$, $y = x$, or $x^2 + y^2 = 1$ in $\mathbb{R}^3$.
- (d) Describe regions of $\mathbb{R}^3$ given by equations and inequalities, and vice versa.
- (e) Compute the distance between two points in $\mathbb{R}^3$.
- (f) Find the equation of a sphere from geometric information (for instance, a center and a point on the sphere, or a sphere tangent to a coordinate plane). Find the center and radius of a sphere by completing the square.

3. Vectors (12.2):

- (a) Know the geometric and algebraic definitions of vector. Know the difference between a point and a vector.
- (b) Compute displacement vectors $\overrightarrow{AB}$ and position vectors algebraically and geometrically.
- (c) Compute sums, differences, and scalar multiples of vectors, both algebraically (i.e. using components) and geometrically (i.e. tip-to-tail method and parallelogram pictures).
- (d) Know the geometric interpretation of $\mathbf{u} - \mathbf{v}$ (the vector points from the tip of $\mathbf{v}$ to the tip of $\mathbf{u}$).
- (e) Compute the magnitude of a vector, and use the formula $|c\mathbf{v}| = |c| |\mathbf{v}|$.
- (f) Know the definition of unit vector. Find a unit vector in the same or opposite direction as a given vector. Write a vector as a sum of scalar multiples of the standard unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.
- (g) Determine when two vectors are parallel using the definition (one is a scalar multiple of the other).
- (h) Know the triangle inequality and when equality holds.

4. Dot product (12.3):

- (a) Know both the algebraic and geometric formulas for the dot product and how to compute it.
- (b) Know the algebraic properties of the dot product (such as $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$) and use them to compute quantities such as $|\mathbf{u} - 2\mathbf{v}|$ from $|\mathbf{u}|$, $|\mathbf{v}|$, and $\mathbf{u} \cdot \mathbf{v}$.
- (c) Know the definition of the angle between two vectors. Find the cosine of an angle between two vectors using the dot product.
- (d) Determine whether the angle between two vectors is acute, obtuse, or a right angle using the sign of the dot product. Determine when two vectors are orthogonal.
- (e) Find the direction cosines and direction angles of a vector.
- (f) Compute the scalar projection $\text{comp}_{\mathbf{u}} \mathbf{v}$ and the vector projection $\text{proj}_{\mathbf{u}} \mathbf{v}$, and split $\mathbf{v}$ into a part parallel to $\mathbf{u}$ and a part orthogonal to $\mathbf{u}$. Know what they mean geometrically.

5. Cross product (12.4):

- (a) Compute a determinant of a 2×2 or 3×3 matrix and interpret the result geometrically (signed area/signed volume).
- (b) Compute the cross product of two vectors using the algebraic definition (the determinant formula).
- (c) Know and use the geometric description of the cross product: $\mathbf{u} \times \mathbf{v}$ is the unique vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ whose length is $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}| \sin \theta$ (the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$) and whose direction makes $\mathbf{u}, \mathbf{v}, \mathbf{u} \times \mathbf{v}$ a right-handed triple.
- (d) Compute the area of a parallelogram or triangle using the cross product, given vectors or vertices.
- (e) Know the algebraic properties of the cross product, including $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$, $\mathbf{u} \times \mathbf{v} = \mathbf{0}$ exactly when $\mathbf{u}$ and $\mathbf{v}$ are parallel, and the cross product is not associative.
- (f) Compute the scalar triple product, and use it to find the volume of a parallelepiped.
- (g) Determine when three vectors or four points are coplanar.

6. Lines and planes (12.5):

- (a) Find a vector equation, parametric equations, and symmetric equations for a line, given a point and a direction or given two points.
- (b) Determine whether a given point lies on a line.
- (c) Find the equation of a line from various geometric information. For instance, a line through a

- point and parallel to a given line, or a line through a point and perpendicular to a given plane.
- (d) Determine whether two lines are parallel, intersect, or are skew. Find the point of intersection when they intersect.
 - (e) Find a vector equation and a scalar equation for a plane given a point and a normal vector.
 - (f) Find the equation of a plane from various geometric information. For instance, a plane containing three specified points, a plane containing a line and a point, or a plane containing a point and perpendicular to two given planes.
 - (g) Determine when two planes are equal, parallel, or orthogonal, and find the angle between two planes.
 - (h) Find the line of intersection of two planes that are not parallel, and find the point where a line meets a plane.
 - (i) Find the distance from a point to a plane.

7. Vector functions and space curves (13.1):

- (a) Know what a vector function is, and find its domain.
- (b) Compute limits of vector functions componentwise, and determine where a vector function is continuous.
- (c) Sketch a space curve, and determine which surface a curve lies on.
- (d) Parametrize the curve of intersection of two surfaces.

8. Derivatives and integrals of vector functions (13.2):

- (a) Know the definition of the derivative of a vector function, and compute derivatives componentwise using differentiation rules from Calc I.
- (b) Know that $\mathbf{r}'(a)$ is the tangent vector to the curve traced by $\mathbf{r}(t)$ at the point $\mathbf{r}(a)$, provided that $\mathbf{r}'(a) \neq \mathbf{0}$.
- (c) Find the unit tangent vector $\mathbf{T}(t)$.
- (d) Find the tangent line to a curve at a specified point.
- (e) Use the differentiation rules, including the product rules for dot and cross products.
- (f) Know that if $|\mathbf{r}(t)|$ is constant, then $\mathbf{r}'(t)$ is orthogonal to $\mathbf{r}(t)$.
- (g) Compute indefinite and definite integrals of vector functions componentwise, using antiderivatives and the Fundamental Theorem of Calculus.
- (h) Recover $\mathbf{r}(t)$ from $\mathbf{r}'(t)$ together with an initial value.

9. Motion in space (13.4):

- (a) Find the velocity, speed, and acceleration of an object from the vector-valued function describing its position.
- (b) Find the velocity and position of an object from the vector-valued function describing its acceleration and initial conditions.